Can Quantum Cosmology Give Observational Consequences of Many-Worlds Quantum Theory?

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Abstract.

Although many people have thought that the difference between the Copenhagen and many-worlds versions of quantum theory was merely metaphysical, quantum cosmology may allow us to make a physical test to distinguish between them empirically. The difference between the two versions shows up when the various components of the wavefunction have different numbers of observers and observations. In the Copenhagen version, a random observation is selected from the sample within the component that is selected by wavefunction collapse, but in the many-worlds version, a random observation is selected from those in all components. Because of the difference in the samples, probable observations in one version can be very improbable in the other version.

INTRODUCTION

Ever since Hugh Everett III formulated his many-worlds alternative [1,2] to the Copenhagen version of quantum theory, there has been considerable discussion of its merits. Many people, including some of the original supporters of the many-worlds version, have expressed the opinion that the many-worlds version is empirically indistinguishable from the Copenhagen version, so that the difference is merely metaphysical.

For example, in the first wide popularization of the many-worlds or Everett-Wheeler-Graham (EWG) version of quantum theory, Everett's bulldog Bryce De-Witt stated [3], "Clearly the EWG view of quantum mechanics leads to experimental predictions identical with those of the Copenhagen view."

Everett's Ph.D. supervisor John Wheeler, who initially supported the many-worlds version [4], has recently summarized it as follows [5]: "Does it offer any new insights? Does it predict outcomes of experiments that differ from outcomes

predicted in conventional quantum theory? The answer to the first question is emphatically yes. The answer to the second question is emphatically no."

Roland Omnès, though never a supporter of the many-worlds version to my knowledge, has thought about it deeply and concluded [6], "If quantum mechanics were absolutely true and Everett were right, no experiment would be able to confirm or reject it. ...It is not science because no experiment can show it to be wrong."

However, David Deutsch has argued [7] that the many-worlds version of quantum theory would be confirmed if an observer could "split" into two copies which make different observations, remember that they observed but not what they observed, and then are rejoined coherently. Although there are conceptual loopholes (such as claiming after the experiment that the observer's memory of having made a definite observation is merely a false memory), I believe this argument is fairly strong evidence that the difference between the Copenhagen and many-worlds versions of quantum theory is, in principle at least, a matter that could be experimentally tested. Nevertheless, this proposed test appears to be technically very difficult.

Because of the difficulties of Deutsch's proposed experiment, here I wish to raise the possibility that quantum cosmology might in principle lead to empirical distinctions between the Copenhagen and the many-worlds versions of quantum theory.

By the Copenhagen version, I essentially mean what I might more accurately call a single-history version, in which quantum theory gives probabilities for various alternative sequences of events, but only one sequence actually occurs. Each such alternative sequence might be called a "history" or a "world."

In the many-worlds version, in contrast, all of the possible histories or worlds with nonzero quantum probabilities actually occur, with the quantum probabilities being not probabilities for the histories to be actualized (since all are), but instead essentially measures for the magnitude of the existence of the various histories.

CONSEQUENCES OF DIFFERENT NUMBERS OF OBSERVERS

There can be significant differences in typical observations if the number of observers varies greatly from "world" to "world." Consider the following toy models:

Quantum Cosmology Model I

World 1: Observers; measure or probability 10^{-100}

World 2: No observers; measure or probability $1 - 10^{-100}$

In a single-history version of this Model I, World 1 is very improbable to occur at all, so any observation would be strong evidence against the single-history version. In a many-worlds version, World 1 does occur, so observations are not evidence against that theory.

Quantum Cosmology Model II

World A: 10^{10} observers during collapse; measure $1 - 10^{-30}$

World B: 10^{90} observers during expansion; measure 10^{-30}

In a single-history version of Model II, World B is very improbable, so a random observation should expect to see a collapsing universe, Hubble constant $H < 0$, and the probability that $H > 0$ is observed is only 10^{-30}.

In contrast, in a many-worlds version of Model II, all of the observations occur, with measures presumably given by something like the expectation values of positive operators each associated with a corresponding observation [8,9]. I shall assume that the observers are sufficiently similar that the total measure of a certain set of observations in a certain world (e.g., of whether the universe is expanding) is roughly proportional to the total number of observers in that world who make the observation, multiplied by the quantum measure of that world. I shall also assume that the fraction of observers who do observe whether the universe is expanding or contracting is the same in both World A and World B.

Then the total measure for World A observations of a collapsing universe is roughly proportional to the 10^{10} observers times the quantum measure of nearly unity for that world, or 10^{10}, whereas the total measure for World B observations of an expanding universe is roughly proportional to 10^{90} observers of that world times the quantum measure of 10^{-30} for that world, or 10^{60}. Thus a random observation chosen from the sample of all existing observations in the many-worlds version is about 10^{50} times more likely to be from World B, seeing $H > 0$, than it is to be from World A, seeing $H < 0$, a situation qualitatively the reverse of the relative probabilities in a single-history version, such as the Copenhagen version of quantum theory.

Now if one accepted the basic quantum measures of the two worlds in Quantum Cosmology Model II but was not very certain whether a single-history or a many-worlds version of quantum theory were correct, then if one made an observation of whether the universe were expanding or contracting, it would give strong evidence as to which version is correct.

One way to explain the difference between sampling a random observation in single-history versus many-worlds quantum theories is with lottery tickets. Suppose that we have a quantum cosmological model with the following two worlds:

World 1: N_1 observers; quantum measure or probability p_1
World 2: N_2 observers; quantum measure or probability p_2

The single-history version of quantum theory is like assigning lottery tickets to World 1 and World 2 in the ratio $p_1 : p_2$. Then a lottery ticket is chosen at random to select which world, and its observers, exist.

The many-worlds version of quantum theory is like assigning lottery tickets to each observer in World 1 and 2 with ratio $p_1 : p_2$, so that the ratio of the total number of lottery tickets in world 1 to that in world 2 is $N_1 p_1 : N_2 p_2$. All the observers exist, but with different measures for their reality, analogous to holding different numbers of lottery tickets. Choosing a measure-weighted observer (or, better, observation) at random is analogous to choosing a lottery ticket at random. The choice really is not made (since all observations really exist in the many-worlds version), but for saying which observations are typical, is is helpful to imagine their being chosen randomly.

PRELIMINARY EVIDENCE FROM HARTLE-HAWKING

We cannot yet calculate probabilities for our observations from an accepted model of the quantum cosmology quantum measures, so we cannot yet perform a definitive test of whether the single-history or the many-worlds version of quantum theory is correct. However, we can examine some highly speculative preliminary suggestions from the Hartle-Hawking 'no-boundary' proposal [10–12] applied to a $k = +1$ Friedmann-Robertson-Walker model with a minimally coupled massive scalar field (potential $\frac{1}{2}m^2\phi^2$).

In this minisuperspace model, an approximation of the stationary phase approximation for the path integral in which the scalar field starts at a value ϕ_i large compared with the Planck value (unity here) leads to the universe nucleating with initial size

$$a_i^2 = \frac{3}{4\pi m^2 \phi_i^2} = \frac{p}{\pi} \tag{1}$$

and quantum measure roughly proportional to

$$e^{-2S_E} \approx e^{\pi a_i^2} = e^p \tag{2}$$

with $p \equiv \pi a_i^2$. Observations suggest $m \sim 10^{-6}$ [13].

This is actually a measure density, and it is not clear what the prefactor should be. One simple choice is $dp = 2\pi a_i da_1$. The resulting measure would diverge if integrated to $p = \infty$ or $a_i = \infty$, but this would correspond to $\phi_i = 0$, where the approximation is invalid. To get an inflationary solution, one needs $\phi_i > \phi_{\min} \sim 1$, so $a_i < a_m = \sqrt{3/4\pi}/(m\phi_{\min}) \sim 1/m$ or $p < p_m = 3/(4m^2\phi_i^2) \sim 1/m^2$. Cut off the measure density there and normalize it, so we get the simple idealization

$$P(p < p') \approx \frac{e^{p'} - 1}{e^{p_m} - 1} \approx e^{-p_m}(e^{p'} - 1) \tag{3}$$

for $p' \equiv \pi a_i^2 < p_m \equiv \pi a_m^2 \sim 1/m^2 \sim 10^{12} \gg 1$.

After the universe nucleates, it undergoes slow-roll inflation with ϕ decreasing from ϕ_i to $\phi_e \sim 1$ and the volume increasing to

$$V_e = V_i \left(\frac{a_e}{a_i}\right)^3 \approx \frac{\sqrt{27\pi}}{4m^3\phi_i^3}e^{6\pi(\phi_i^2-\phi_e^2)} = 2\sqrt{\pi}e^{-6\pi\phi_e^2}p^{3/2}\exp\frac{4.5\pi}{m^2p} \sim p^{3/2}\exp\frac{4.5\pi}{m^2p}, \tag{4}$$

which implies that for $m^3 V_e \gg 1$,

$$p \sim \frac{4.5\pi/m^2}{\ln(m^3 V_e) + 1.5\ln\ln(m^3 V_e)}. \tag{5}$$

The entropy density after reheating is

$$s_e \sim T_e^3 \sim \rho_e^{3/4} \sim (m^2 \phi_e^2)^{3/4} \sim m^{3/2} \sim 10^{-9}. \tag{6}$$

By comparison, the entropy density of radiation today is

$$s_0 \approx \frac{86\pi^2}{165} T_0^3 \approx 1.22 \times 10^{-95}. \tag{7}$$

Assuming essentially adiabatic expansion after reheating, one gets that the volume of the universe today is

$$V_0 \approx \frac{s_e}{s_0} V_e \sim 10^{95} m^{3/2} V_e \sim 10^{95} m^{3/2} p^{3/2} \exp\frac{4.5\pi}{m^2 p} \sim 10^{86} p^{3/2} e^{1.4\times10^{13}/p}. \tag{8}$$

Now to get something analogous to Quantum Cosmology Model II above, we need to consider what values of p give observers mainly seeing the universe either contracting or expanding, and how many observers are produced as a function of p.

Let us make the crude assumption that observers require a universe of an age at least of the order of 10^{60}, a tenth of the age of our actual universe, and hence a volume of the order of 10^{181}, in order for suitable habitats to have evolved (e.g., planets around stars). This would give a lower limit on the volume at the end of inflation of about

$$V_e \gtrsim 10^{86} m^{-3/2} \sim 10^{95}. \tag{9}$$

Inserting this back into the approximate relation between V_e and p gives

$$p < p_{\text{max}} \sim \frac{4.5\pi/m^2}{\ln\left(10^{77}\right) + 1.5 \ln\ln\left(10^{77}\right)} \sim \frac{4.5\pi/m^2}{185} \sim 7.6 \times 10^{10} \tag{10}$$

as the crude condition for the existence of observers.

However, if p is sufficiently near this upper limit p_{max} for the existence of observers, then the universe will just barely last long enough for them, and they will mostly exist near the end of the lifetime of the universe, when it is collapsing. For most observers to see the universe expanding, V_e must be sufficiently larger that the lifetime of the universe is long enough for most observers to exist while the universe is still expanding. If the present age of the universe is a typical time for observers, then one might estimate that the universe must still be expanding at an age of roughly 10^{61} for most observers to see the universe expanding, and hence for it to have a volume of at least of the order of 10^{184} then. This leads us to $V_e \gtrsim 10^{89} m^{-3/2} \sim 10^{98}$ and

$$p < p_{\text{exp}} \sim \frac{4.5\pi/m^2}{\ln\left(10^{80}\right) + 1.5 \ln\ln\left(10^{80}\right)} \sim \frac{4.5\pi/m^2}{192} \sim 7.4 \times 10^{10} \tag{11}$$

as the crude condition for most observers to see the universe expanding.

In other words, in the Hartle-Hawking minisuperspace model under consideration, if $0 < p < p_{\mathrm{exp}} \sim 7.4 \times 10^{10}$, observers will exist and will mostly see the universe expanding; if $p_{\mathrm{exp}} < p < p_{\mathrm{max}} \sim 7.6 \times 10^{10}$, observers will exist but will mostly see the universe contracting; and if $p_{\mathrm{max}} < p$, essentially no observers will exist.

First, consider a Copenhagen or other single-history version of this quantum minisuperspace model, in which the wavefunction collapses to give a single macroscopic history or world, a classical Friedmann-Robertson-Walker universe characterized by ϕ_i, a_i, or p.

Using the results above, the probability for the wavefunction to collapse to classical universe that lasts long enough for observers is

$$P(\text{observers}) \approx \frac{e^{p_{\mathrm{max}}} - 1}{e^{p_m} - 1} \approx e^{p_{\mathrm{max}} - p_m} \sim 10^{-401000000000}, \qquad (12)$$

and the probability for it to have observers that mostly see the universe expanding is

$$P(\text{observers seeing expansion}) \approx e^{p_{\mathrm{exp}} - p_m} \sim 10^{-402000000000}, \qquad (13)$$

both of which are utterly tiny.

Therefore, unless one had an uncertainty less than roughly $e^{-0.924 \times 10^{12}} \sim 10^{-40100000000}$ that this single-history model was correct, the evidence that observers exist would be overwhelming evidence against it.

Even if one somehow claimed that observers were necessary (i.e., that the wavefunction could not collapse to a world with no observers), the conditional probability of a world with observers mostly seeing the universe expand, given the condition that observers exist, is only

$$P(\text{observers seeing expansion}|\text{observers exist}) \approx e^{p_{\mathrm{exp}} - p_{\mathrm{max}}} \sim 10^{-1000000000}. \quad (14)$$

Thus the observation that the universe is expanding would be strong evidence against the single-history version of this model.

On the other hand, if one takes a many-worlds version of this Hartle-Hawking minisuperspace model, all components of the wavefunction exist that have positive measure, no matter how small, so observers will exist in a generalization of the model that allows sufficient structure for observers. Therefore, the existence of observers would not be evidence against a many-worlds version of a model sufficiently general to allow observers within at least some components of the wavefunction.

However, we can still ask for the relative probabilities of observations that the universe is contracting or expanding. In a many-worlds version, this will be roughly proportional to the bare quantum probability for each world multiplied by the the number of observers for that world. The unnormalized bare quantum probability was given above as $dP \approx e^p dp$ for $0 < p < p_m \sim 1/m^2 \sim 10^{12}$. For $0 < p < p_{\mathrm{max}}$, observers exist, and it is reasonable to assume that the number of them is very

roughly proportional to the volume V_0 of the universe when the entropy density is roughly the value of 10^{-95} that we observe, assuming that our observations of this quantity are typical. Then in this range of p, we get an unnormalized observational probability density roughly proportional to

$$dP_{\text{obs}} \approx V_0 dP \approx V_0 e^p dp \sim 10^{95} m^{3/2} p^{3/2} \exp\left(\frac{4.5\pi}{m^2 p} + p\right) dp. \qquad (15)$$

The integral of this over $0 < p < p_{\max}$ diverges for $p \to 0$ ($V_0 \to \infty$), because of the divergence in the number of observers there, so effectively all of the observational probability occurs at that limit (infinitely large universes with infinitely many observers, presumably almost all seeing the universe expanding, since the stars would have all burned out in the infinite time it takes the infinitely large universe to recollapse.)

Therefore, in a many-worlds version of this Hartle-Hawking quantum cosmological model, one would presumably expect with very nearly unit probability that a random observation would see the universe expanding, the opposite of one's expectation for a single-history version of the same model. Thus if one accepted the basic model and allowed at least some reasonable uncertainty as to whether a single-history or a many-worlds version of the model is correct (before considering the evidence of the sign of the Hubble constant), then one's observation of whether the universe is expanding or contracting would give very strong evidence in support of either the many-worlds or the single-history version respectively. This is very similar qualitatively to the toy Quantum Cosmology Model II discussed above, except here the quantitative differences are even grossly more severe.

Of course, this preliminary evidence from a particular implementation of the Hartle-Hawking no-boundary proposal is highly speculative and is meant to be mainly illustrative, because of the many uncertainties of the model.

CONCLUSIONS

If the amount of observations (roughly, the number of observers) varies for different wavefunction components, then observation probabilities depend on whether only one component occurs in actuality (a single-history version of quantum theory, where observations truly are made only in one history, world, or component of the wavefunction), or whether many do (a many-worlds version, where observations truly are made in many histories, worlds, or components of the wavefunction). In particular, if components with relatively few observations dominate the quantum amplitude, but other components with testably different observations dominate the expectation value of the number of observations, which observations are most probable varies between single-history and many-worlds quantum theories.

The Hartle-Hawking wavefunction might allow a test from the observed expansion of the universe, but as of now it is highly speculative whether it is correct and what relative probabilities it would give for observing the universe expanding.

A shorter version of this paper has been circulated [14] and has been reported on in the lay literature [15].

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